**Supplementary Information**

**Sessile Droplet Jumping Behaviors on Hot Substrates:**

**from Vibration to Explosion**

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Diagram

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**Supplementary Figure 1. Experimental setup for the droplet jumping study.** A water microdroplet is initially deposited on the micro-pillared substrate staying in Wenzel state. Then the substrate with the sessile droplet is carefully translated to the preheated hot plate at steady state for boiling jumping observation. The temperature of the substrate is measured by a K-type thermocouple embedded in the hot plate underneath the substrate. And surface temperature of the droplet is monitored by an IR camera (FLIR A655sc) after moving the substrate on the hot plate. The vibration and jumping processes of the boiling droplet are recorded by a side view high-speed camera (nac MEMRECAM HX-3) and the vapor bubble growth process is monitored by another high-speed camera from the top (Photron FASTCAM SA-6).

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**Supplementary Figure 2. Substrate topology and droplet contact angles on different substrates.** (a) Scanning electron micrography (SEM) of substrate . (b) SEM image of substrate . (c) SEM image of substrate . (d) SEM image of substrate . (e) Relationship between the substrate roughness and the advancing contact angle (ACA) / receding contact angle (RCA) of droplets in the Cassie state. (f) Relationship between the substrate roughness and the advancing/receding contact angles of droplets in the Wenzel state.

It is observed in Supplementary Fig. 2(e) that, for droplets in Cassie state, both the ACA and RCA are independent of the substrate roughness. In this study, the increase of the substrate roughness is mainly caused by the increase of the micropillar height *H*. The Cassie state droplet residues only on the top surface of the micropillars and as a result, the increase of the substrate roughness has no obvious influence on the Cassie state droplet ACA and RCA.

However, it is observed in Supplementary Fig. 2(f) that the ACA of droplet in Wenzel state increases with the increase of the substrate roughness and the RCA of the droplet in Wenzel state decreases with the increase of the substrate roughness. The increase of the micropillar height (increase of the substrate roughness) leads to the increase of the adhesion between the Wenzel state droplet and the substrate. Thus, it requires a larger ACA or a smaller RCA to generate a depinning force to achieve the movement of the pinned droplet contact line.

ACA and RCA of droplets in both Cassie state and Wenzel state are measured and exhibited in Supplementary Table 1. To measure the ACA, the droplet is deposited on the substrate with a syringe needle left inside the droplet bulk. The water liquid is slowly and steadily pumped into the droplet to increase the volume of the droplet. Simultaneously, the droplet contact angle increases while the contact line of the keeps pinned. The transient contact angle is recorded as the ACA when the droplet contact line begins to move. The opposite is done to measure the RCA: water liquid is pumped out from the droplet and the transient contact angle is recorded as the RCA when the droplet contact line starts to shrink. One should note that the RCA usually depends on the size of the droplet. The measured RCA might not be accurate when the sessile droplet volume is too small. Korhonen *et al*.[1] gave a reasonable lower limit of the droplet volume as 5µL to measure a reliable RCA of a sessile water droplet. Thus, in this study, all the RCAs are measured with droplet volume larger than 5 µL.

**Supplementary Table 1.** Experimental values of advancing contact angle (ACA) and receding contact angle (RCA) on the silicon-based micropillared substrates used in this study

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Specifications** | **Cassie state ACA (°)** | **Cassie state RCA (°)** | **Wenzel state ACA (°)** | **Wenzel state RCA (°)** |
| [*D*, *L*, *H*] = [20, 120, 20] µm | 161±2 | 141±1 | 135±2 | 103±2 |
| [*D*, *L*, *H*] = [20, 120, 40] µm | 160±2 | 138±3 | 136±4 | 92±2 |
| [*D*, *L*, *H*] = [20, 120, 60] µm | 158±2 | 139±3 | 131±2 | 78±2 |
| [*D*, *L*, *H*] = [20, 120, 80] µm | 158±2 | 146±1 | 135±2 | 77±9 |
| [*D*, *L*, *H*] = [20, 140, 20] µm | 161±1 | 143±3 | 134±2 | 106±1 |
| [*D*, *L*, *H*] = [20, 140, 40] µm | 161±3 | 146±1 | 135±2 | 102±4 |
| [*D*, *L*, *H*] = [20, 140, 60] µm | 159±2 | 148±1 | 136±3 | 86±4 |
| [*D*, *L*, *H*] = [20, 140, 80] µm | 159±1 | 147±2 | 141±3 | 75±2 |
| [*D*, *L*, *H*] = [20, 160, 20] µm | 159±2 | 147±4 | 134±1 | 108±2 |
| [*D*, *L*, *H*] = [20, 160, 40] µm | 160±2 | 141±2 | 136±2 | 95±3 |
| [*D*, *L*, *H*] = [20, 160, 60] µm | 160±2 | 145±2 | 135±1 | 83±2 |
| [*D*, *L*, *H*] = [20, 160, 80] µm | 159±1 | 146±1 | 134±2 | 80±3 |
| [*D*, *L*, *H*] = [30, 120, 20] µm | 160±1 | 139±1 | 139±1 | 98±1 |
| [*D*, *L*, *H*] = [30, 120, 40] µm | 159±2 | 142±3 | 143±1 | 97±2 |
| [*D*, *L*, *H*] = [30, 120, 60] µm | 159±2 | 135±4 | 139±1 | 73±2 |
| [*D*, *L*, *H*] = [30, 120, 80] µm | 157±2 | 140±5 | 148±1 | 77±5 |
| [*D*, *L*, *H*] = [30, 140, 20] µm | 159±1 | 145±2 | 139±1 | 108±4 |
| [*D*, *L*, *H*] = [30, 140, 40] µm | 158±2 | 143±1 | 143±4 | 102±3 |
| [*D*, *L*, *H*] = [30, 140, 60] µm | 159±4 | 140±4 | 140±4 | 85±1 |
| [*D*, *L*, *H*] = [30, 140, 80] µm | 160±1 | 144±2 | 145±3 | 75±1 |
| [*D*, *L*, *H*] = [30, 160, 20] µm | 161±1 | 147±1 | 135±2 | 107±4 |
| [*D*, *L*, *H*] = [30, 160, 40] µm | 158±1 | 143±3 | 147±2 | 104±3 |
| [*D*, *L*, *H*] = [30, 160, 60] µm | 160±2 | 137±3 | 145±2 | 85±3 |
| [*D*, *L*, *H*] = [30, 160, 80] µm | 160±2 | 143±2 | 142±1 | 83±6 |
| [*D*, *L*, *H*] = [40, 120, 20] µm | 159±2 | 132±5 | 149±2 | 96±2 |
| [*D*, *L*, *H*] = [40, 120, 40] µm | 160±2 | 134±4 | 160±2 | 102±1 |
| [*D*, *L*, *H*] = [40, 120, 60] µm | 160±2 | 121±5 | 145±4 | 65±4 |
| [*D*, *L*, *H*] = [40, 120, 80] µm | 159±1 | 139±4 | 151±1 | 62±8 |
| [*D*, *L*, *H*] = [40, 140, 20] µm | 157±1 | 143±2 | 140±1 | 102±2 |
| [*D*, *L*, *H*] = [40, 140, 40] µm | 159±2 | 139±4 | 150±2 | 97±2 |
| [*D*, *L*, *H*] = [40, 140, 60] µm | 160±2 | 141±1 | 149±4 | 84±1 |
| [*D*, *L*, *H*] = [40, 140, 80] µm | 159±2 | 143±3 | 149±3 | 73±4 |
| [*D*, *L*, *H*] = [40, 160, 20] µm | 160±2 | 142±1 | 138±1 | 108±1 |
| [*D*, *L*, *H*] = [40, 160, 40] µm | 160±2 | 142±3 | 142±2 | 96±3 |
| [*D*, *L*, *H*] = [40, 160, 60] µm | 159±1 | 139±3 | 146±2 | 88±3 |
| [*D*, *L*, *H*] = [40, 160, 80] µm | 160±3 | 146±1 | 147±2 | 87±4 |

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**Supplementary Figure 3. Heat transfer enhancement of micropillar array.** (a) droplet resting on substate heated at 130 °C. (b) droplet resting on substate heated at 130 °C.

For droplet in Wenzel state sitting on a micropillared substrate, the micropillars penetrate the droplet base and functions as fin array for heat transfer modulation. Supplementary Fig. 3 presents the diagram of Wenzel state droplets on hot substrates with different micropillar heights.The micropillared substrates consist of micropillars with the same diameter (), periodicity () but with different micropillar heights ( in Supplementary Fig. 3a and in Supplementary Fig. 3b). Considering the steady state convection heat transfer between the Wenzel state droplet and the substrate, the energy balance for the substrate can be given by:

(1)

where is the heat transfer in rate from the hot plate to the substrate and is the heat transfer out rate from the substrate to the liquid droplet.

We can divide the heat transfer rate to two parts: one part is the heat transfer rate from the micropillar fins to the liquid and the other part is the heat transfer rate from the substrate base to the liquid. The heat transfer out rate can be rewritten as:

(2)

where is the heat transfer rate from a single micropillar fin, is the number of micropillar fins underneath the droplet, and is the heat transfer rate from the substrate base.

The heat transfer rate from an individual micropillar fin is calculated as[2]:

(3)

(4)

(5)

where is the height of the micropillars, is the water convection heat transfer coefficient, is the thermal conductivity of silicon, is the temperature at the micropillar (fin) base, is the water temperature at the droplet base between the micropillars, is the cross-section perimeter of the micropillar fin and is the cross-section area of the micropillar fin.

The heat transfer rate from the substrate base can be calculated as:

(6)

where is the contact area of the droplet with substrate base and is the all the tip area of the micropillars covered by the droplet.

Then we estimate the heat transfer enhancement caused by the increase of the micropillar height. Assuming the diameter of the contact area between the droplet and the substate is , the number of the micropillar fins can be estimated as . The convection heat transfer coefficient can be estimated as 500[2] and the silicon thermal conductivity is . The maximum water temperature measured by the IR camera at the droplet base contact line is about 105 °C. Due to the temperature continuity at the liquid-solid interface, it is reasonable to assume that the base temperature of the substrate °C. The liquid temperature at the droplet base is assumed to be at its saturation temperature °C. With these parameters obtained, we can calculate the heat transfer rate on the substrates with different micropillar heights ( and ).

The heat flux from the substrate of to the droplet is calculated as and from the substrate of to the droplet is In conclusion, the rise of the micropillar height from would cause 35.9% increase of the heat transfer rate.

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**Supplementary Figure 4.** (a) Droplet volume evolution during the vibration jumping process on substrate at 130 °C. (b) Droplet height variation during the vibration jumping process. The droplet volume (mass) loss during the whole vibration jumping process was small. In particular, the droplet volume during the periodic vibration ( - ) was maintained at an almost constant level of . The shadowed region is the droplet vibration phase. Due to the capillary waves on the droplet cap surface, there is an uncertainty for the Wenzel state droplet since we assume the droplet is always in an axisymmetric shape.

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**Supplementary Figure 5. Diagram of mass-spring-damper model for** **vibrating droplet and comparison of the experimental and theoretical results of droplet height.** (a) Diagram of droplet mass-spring system. The vibrating droplet can be modelled as two masses connected with a spring. is the damping constant, is the spring constant, is the initial height of the upper mass . (b) Diagram of droplet prolate-to-oblate ellipsoid transformation. (c) Comparison of experimentally measured droplet height on substrate at 130 °C with theoretical predictions. The theoretical prediction (red line) matches well with the experimental results (black cycles). The shadowed region is the droplet vibration phase.

The vibrating droplet can be modelled as a mass-spring-damper system[3,4] with an external force due to the droplet base depinning as shown in Supplementary Fig. 5. The system can be expressed based on the vertical position of the upper mass as:

(7)

where is the droplet mass, is the position of the upper mass, is the ratio of the upper mass to the whole system, is time, is the damping constant, is the spring constant[3], is the initial height of the upper mass, is the external force. The vibration equation can be rewritten in a non-dimensional form:

(8)

where and is the radius of the droplet, and is the inertial-capillary timescale, is the damping ratio, is the Bond number.

The droplet vibration on the substrate is essentially asymmetric: the droplet downward motion impacts the substrate inhibiting the depinning of the droplet contact line which hampers the external momentum transfer into the droplet, whereas the droplet upward motion has less influence on the contact line depinning. The asymmetric droplet vibration indicates that the momentum received from the depinning process during the upward motion to propel the droplet is more than the momentum obtained during the downward motion to impede the droplet motion. The asymmetric vibration of the droplet leads to the net momentum earning for the droplet in each cycle. As a result, the droplet gradually accumulates kinetic energy from the asymmetric vibration.

Due to the asymmetric mode of the vibration, the external forces acted on the droplet during the downward and upward vibration are different:

(9)

where and are two different non-dimensional constants with . We tuned the external force and the upper mass ratio in the vibration equation to fit the experimental data (two degrees of freedom). The comparison of the theoretical calculation results and the experimental data are shown in Supplementary Fig. 5.

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**Supplementary Figure 6. Droplet temperature evolution and droplet height variation on the hot substrates.** (a) Transient droplet bulk and base temperature evolutions on the hot plate at 130 °C for 7 L water droplets on two substrates and . (b) Transient droplet bulk and base temperature evolutions on the hot plate at 150 °C for water droplets on two substrates and . (c)Transient droplet bulk and base temperature evolutions on the hot plate at 170 °C for water droplets on two substrates and . (d) Droplet height variation on substrate at 130 °C. (e) Droplet height variation on substrate at 140 °C. (f) Droplet height variation on substrate at 150 °C. (g) Droplet height variation on substrate at 160 °C. (h) Droplet height variation on substrate at 170 °C. Droplet is deposited on the hot substrate in Wenzel state and no capillary waves are observed in phase 1. Then vapor bubble bursts at the droplet base and leads to the depinning of droplet contact line. Capillary waves show up on the droplet cap surface in phase 2. But there is no obvious period for droplet vibration during this phase. Specific period of droplet vibration can be recognized in phase 3 and the droplet vibrates with increasing amplitude. Consequently, droplet jumps off the substrate and lands on the surface in Cassie state. The Cassie state droplet vibrates with decaying amplitude in phase 4. Temperatures of the droplet bulk and base on the hot substrates are measured by a calibrated IR camera as shown in the inset of (a). Due to the enhanced heat transfer by the fin-array-like micropillars, the slope of the temporal evolution of droplet temperature on substrate is always larger than that on substrate while the hot plate temperature is maintained at 130 °C, 150 °C and 170 °C, respectively. Though the hot plate temperature increases from 130 °C to 170 °C, the maximum temperature of the droplet base is always maintained below 105 °C. The increase of hot plate temperature leads to a faster droplet temperature rise from 40 °C to ~100 °C. When the base temperature of the droplet exceeds 100 °C, the droplet is apt to jump on substrate or to vibrate on substrate .

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**Supplementary Figure 7. Capillary wave amplitude and wavelength on boiling droplet surface.** (a) Capillary waves formed on the surface of boiling water droplet on substrate at 130 °C (side view). (b) Capillary waves formed on the surface of boiling water droplet on substrate at 130 °C (side view). (c) Capillary waves formed on the surface of boiling water droplet on substrate at 130 °C (side view). (d) Capillary waves formed on the surface of boiling water droplet on substrate at 130 °C (side view). (e) Amplitude and wavelength measurements from the snapshot of boiling water droplet on micropillared substrate. (f) Diagram of a curved surface unit. (g) Relationship between the capillary wave amplitude and the micropillar height. (h) Relationship between the capillary wave wavelength and the micropillar height.

We observed that the boiling droplet surface exhibited different capillary waves on substrates with the same micropillar diameter and periodicity (), at the same substrate temperature (130 °C), but with different micropillar height ( ) as shown in Supplementary Fig. 7. Specifically, the capillary wave amplitude and wavelength are relatively small when the micropillar height is short (). In comparison, droplet capillary waves with relatively larger amplitude and longer wavelength exhibit on a substrate with a relatively taller micropillar height (). We measured the capillary wave amplitude and wavelength (Supplementary Fig. 7e) for water droplets boiling on 32 different substrates and the results are presented in Supplementary Figs. 7g and 7h. It is observed that both the amplitude and the wavelength of capillary waves increase with the increasing substrate micropillar height. However, the micropillar diameter and periodicity have no obvious influence on the capillary wavelength formed on the surface of droplets boiling on the substrates used in this study.

First, we analyzed the total energy of the capillary waves. The total energy of the capillary wave is the summation of its kinetic energy and its potential energy (In general, the average of a wave equal to its average )[5]: . As shown in Supplementary Fig. 7f, the potential energy of the curved capillary wave surface due to the increased surface area in a small unit can be estimated as:

(10)

Assuming the capillary wave is in the form of sinusoidal wave as shown in Supplementary Fig. 7f. , where *A* is the amplitude of the capillary wave, is the wave number, is wavelength, and is the angular frequency. As a result, the average potential energy in a small unit water surface is:

(11)

The total energy of the whole droplet is calculated by integrating over the surface of the droplet:

(12)

In this study, the capillary waves are triggered by the depinning of liquid bridge formed at the droplet contact line. Importantly, we found that the amplitude and the wavelength of the capillary waves mainly depend on the height of the micropillars. Considering the capillary wave dispersion relation, we have . Thus, the relationship between the wavelength and the period is given by , where is the period of the capillary wave. Since the capillary wave is generated by the droplet contact line depinning, the period of the capillary wave is initially determined by the depinning time. It takes a longer time for the liquid bridge (or contact line) to get depinned from the substrate with higher micropillars. Here, the liquid bridge is pined at the bottom of the micropillars, and the pressure difference () pull the liquid bridge upward to finally detach from the tip surface of the micropillars. Therefore, the time for the depinning process can be approximated as: where is the height of the micropillars and is the depinning velocity. Evaporation at the droplet base can increase the pressure inside the vapor bubble and the liquid bridge depinning process occurs when the pressure difference overcomes the adhesion at the bottom of the micropillars. The adhesion at the micropillar base should be in the same level for different substrates due to the same coating material. Thus, when overcoming the adhesion from the micropillar base, the accumulated driving pressure difference should also be in the same level for droplet boiling on different substrates. The depinning velocity depends on the initial energy provided by the pressure difference . Consequently, the depinning velocity on the substrate should be in the same level and is regarded as a constant factor in our analysis. Thus, the relationship between the capillary wavelength and the micropillar height scales as . Then we considered the amplitude of the capillary waves on a boiling droplet surface. The initial amplitude of the capillary waves should be proportional to the length of the liquid bridge, which is determined by the height of the micropillars. Thus, the capillary wave amplitude should be proportional to the micropillar height: . With the relationship between capillary wave amplitude/wavelength and the micropillar height obtained, we can estimate the capillary wave energy stored in droplets boiling on substrate with different micropillar heights. The total capillary wave energy is scaled as:

(13)

Therefore, the capillary wave energy increases with the height of the micropillars.

**Captions of the movies**

**Movie S1**.

**Droplet vibration jumping and explosion jumping on micropillared substrates at 130 °C.**

Side-view videos of the vibration jumping and the explosion jumping, and top-view videos of their corresponding vapor bubble growth processes shown in Fig. 1. Water droplet of 2 mm in diameter resting on substrate starts to boil with heat-transfer-controlled bubble growth while vibrating on the substrate to achieve the out-of-plane jumping. Water droplet of 2 mm in diameter resting on substrate starts to boil with inertia-controlled bubble growth and explosively jumps off the substrate in milliseconds.

**Movie S2**.

**Inertia-controlled bubble growth on hot substrates with large micropillar height.**

Top-view videos of inertia-controlled bubble growth at droplet base on 16 different kinds of substrates with relatively large micropillar height ( and ). The vapor bubble expanding velocities used in Fig. 2 were measured from these videos.

**Movie S3**.

**Five different jumping modes for the boiling water droplet resting on substrates with different topography.**

Droplet shows vibration jumping mode on substrate ; droplet shows vibration jumping in trampolining mode on substrate ; droplet shows explosion jumping on substrate ; droplet shows Wenzel-to-Cassie wetting state transition on substrate ; and droplet shows capillary jumping on substrate .

**Movie S4**.

**Post-impacting wetting states of boiling water droplet.**

Droplet impacts on substrate in Cassie state and then vibrates with decaying amplitude. Droplet impacts on substrate in Wenzel state and trampolines with increasing maximum height.

**Supplementary References**

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